**Complex Numbers**

In Section 1.4, we learned that some quadratic equations have no real solutions. For instance, the quadratic equation:

has no real solution because there is no real number that can be squared to produce -1. To overcome this deficiency, mathematicians created an expanded system of numbers using the **imaginary unit**, defined as:

where. By adding real number to real multiples of this imaginary unit, we obtain the set of **complex numbers**.

Each complex number can be written in the **standard form, .**

**----------------------------------------------------------------------------------------------------------------------------------------------------------------**

WRITING COMPLEX NUMBERS IN STANDARD FORM

1. 2. 3. 4.

----------------------------------------------------------------------------------------------------------------------------------------------------------------

EQUALITY OF COMPLEX NUMBERS

Two complex numbers below, written in standard form, are **equal** to each other if and only if \_\_\_\_\_\_\_\_\_\_\_ and \_\_\_\_\_\_\_\_\_\_\_\_\_\_.

Examples: Find real numbers and so that the following equations are true.

5. 6. 7.

----------------------------------------------------------------------------------------------------------------------------------------------------------------

The **additive identity element** in the complex number system is zero (the same as the real number system).

In the real number system the **additive inverse** of 2 is \_\_\_\_\_\_. Since.

Similarly, the additive inverse of the complex number is: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

ADDING AND SUBTRACTING COMPLEX NUMBERS

Examples: Perform the addition or subtraction and write the result in standard form.

8. 9. 10.

----------------------------------------------------------------------------------------------------------------------------------------------------------------

EXPLORATION 🡪 POWERS OF

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

What do you notice?

Examples: Simplify the following expressions.

11. 12. 13. 14.

MULTIPLYING COMPLEX NUMBERS

15. 16. 17. 18.

\*\*The product of two complex numbers can be a real number.\*\*

This occurs with pairs of complex numbers of the form: and \_\_\_\_\_\_\_\_\_\_\_\_.

**These numbers are called complex conjugates.**

**--------------------------------------------------------------------------------------------------------------------------------------------------**COMPLEX CONJUGATES AND DIVISION

To find the quotient of complex numbers, and , \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

Examples: Simplify the following quotients in form.

19. 20. 21.

----------------------------------------------------------------------------------------------------------------------------------------------------------------

COMPLEX SOLUTIONS OF QUADRATIC EQUATIONS

Examples: Solve the following quadratic equations in simplest form.

22. 23.