**Complex Numbers**

In Section 1.4, we learned that some quadratic equations have no real solutions. For instance, the quadratic equation:

$$x^{2}+1=0$$

has no real solution because there is no real number $x $that can be squared to produce -1. To overcome this deficiency, mathematicians created an expanded system of numbers using the **imaginary unit**$ i $, defined as:

$$i= \sqrt{-1}$$

where$ i^{2}= -1$. By adding real number to real multiples of this imaginary unit, we obtain the set of **complex numbers**.

 Each complex number can be written in the **standard form,** $a+bi$**.**

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WRITING COMPLEX NUMBERS IN STANDARD FORM

1. $ 5+ \sqrt{-9}$ 2. $ 4+\sqrt{-16}$ 3. $ 3+ \sqrt{-8}$ 4. $1- \sqrt{-27}$

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EQUALITY OF COMPLEX NUMBERS

Two complex numbers below, written in standard form, are **equal** to each other if and only if \_\_\_\_\_\_\_\_\_\_\_ and \_\_\_\_\_\_\_\_\_\_\_\_\_\_.

$$a+bi=c+di$$

Examples: Find real numbers $a$ and $b$ so that the following equations are true.

5. $a+bi= -11+7i$ 6. $a+bi=13-4i$ 7. $\left(a-1\right)+\left(b+3\right)i=6+8i$

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The **additive identity element** in the complex number system is zero (the same as the real number system).

In the real number system the **additive inverse** of 2 is \_\_\_\_\_\_. Since$ \\_\\_\\_\\_\\_+\\_\\_\\_\\_\\_\\_= \\_\\_\\_\\_\\_\\_\\_\\_\\_$.

Similarly, the additive inverse of the complex number $a+bi$ is: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

ADDING AND SUBTRACTING COMPLEX NUMBERS

Examples: Perform the addition or subtraction and write the result in standard form.

8. $\left(3-i\right)+\left(2+3i\right)$ 9. $2i-\left(-4-2i\right)$ 10. $ 3-\left(-2+3i\right)+\left(-5+i\right)$

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EXPLORATION 🡪 POWERS OF $i$

|  |  |  |  |
| --- | --- | --- | --- |
| $$i^{1}=$$ | $$i^{5}=$$ | $$i^{9}=$$ | $$i^{13}=$$ |
| $$i^{2}=$$ | $$i^{6}=$$ | $$i^{10}=$$ | $$i^{14}=$$ |
| $$i^{3}=$$ | $$i^{7}=$$ | $$i^{11}=$$ | $$i^{15}=$$ |
| $$i^{4}=$$ | $$i^{8}=$$ | $$i^{12}=$$ | $$i^{16}=$$ |

What do you notice?

Examples: Simplify the following expressions.

11. $ i^{35}$ 12. $i^{12}+ i^{14}+i^{20}-i^{6}$ 13. $5i^{9}+ i^{25} $ 14. $i^{8 }•2i^{10}•4i^{22}$

MULTIPLYING COMPLEX NUMBERS

15. $\left(i\right)\left(-3i\right)$ 16. $\left(2-i\right)(4+3i)$ 17. $(3+2i)^{2}$ 18. $(3+2i)(3-2i)$

\*\*The product of two complex numbers can be a real number.\*\*

This occurs with pairs of complex numbers of the form: $a+bi $ and \_\_\_\_\_\_\_\_\_\_\_\_.

**These numbers are called complex conjugates.**

$$\left(a+bi\right)\left(a-bi\right)= $$

**--------------------------------------------------------------------------------------------------------------------------------------------------**COMPLEX CONJUGATES AND DIVISION

To find the quotient of complex numbers, $a+bi $ and $c+di$, \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.

Examples: Simplify the following quotients in $a+bi$ form.

19. $\frac{4}{2+i}$ 20. $\frac{5i}{10-i}$ 21. $\frac{2+3i}{4-2i}$

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COMPLEX SOLUTIONS OF QUADRATIC EQUATIONS

Examples: Solve the following quadratic equations in simplest $a+bi $form.

22. $3x^{2}-2x+5= 0 $ 23. $16x^{2}-4t+3=0$